An Algorithm Study for Evaluating Subjective Index of Industrial Design Based on the Fuzzy Theory and the Grey Theory

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Abstract: Based on the fuzzy theory and the grey theory, a new algorithm for evaluating subjective index of industrial design is proposed. The evaluating items are arranged in a hierarchical structure with several levels. The relative contribution (weighting function) of each item to the overall value of the solution and the rating or degree of approximation of a solution with respect to a given item is quantified with the experiential membership functions of a fuzzy set. But it is inevitable to get birth to the deviation that the individual factor brings. The evaluating numerical value is not very exact, so it is fuzzy character and grey character. Then two evaluation models with the fuzzy ratiocination and the grey ratiocination are presented. A computer program built with a weighted generalized mean method is used to calculate the probability level by level from the lowest-level items. After the expected values of the top-level items are calculated, they are then composed with the evidential reasoning approach. The result is used to make a decision quantitatively on selecting the optimal design alternative. Being tested and used in reality, this algorithm is practical and feasibly. The method can also be used to solve other design problems.

Key words: Design evaluation; Industrial design; The fuzzy theory; The grey theory.

1. Introduction

In recent years, design evaluation has gradually become an important research subject of industrial design. But some items, such as era, harmony, cognitive, concordance, rationality and etc., are subjective, they are still unable to objective quantification and evaluated with subjective assessment that is built upon estimators who have different individual knowledge competence, cognitive ability and favorite. Therefore, it is inevitable to get birth to the deviation that the individual factor brings. The evaluation information is neither exact nor complete, but it is fuzzy character and grey character. Although there are some methods of industry design evaluation, such as the evaluation of fuzzy set theory [1][2], the evaluation of mathematical statistics [3], the evaluation of weight coefficient [4], the evaluation of administrative levels analyzing [5], etc.. No matter what kind of appraisal method, it fails to settle two problems. Firstly, the grey information in the design evaluation is not got rid of. Secondly, a lot of level appraise result are not synthesized rationally. This paper presents a new algorithm for evaluating subjective index of industrial design based on the fuzzy ratiocination and the grey ratiocination. A computer program built with a weighted generalized mean method is used to calculate the probability level by level from the lowest-level items. After the expected values of the top-level items are calculated, they are then composed with the evidential reasoning approach. The result is used to make a decision quantitatively on selecting the optimal design alternative.

2. Evaluating algorithm of industry design

2.1 Evaluating model of industry design

According to evaluation content of the industry design, all main items can be graded firstly. All factors, $F = \{F_i\}(i = 1,2,\cdots,n)$, are composing of the total factor set, where F_i is separately corresponding to the kind of main characteristics factor and is called the first-class factor. Every main factor is composed of second-class factor. The i main characteristic factor F_i is divided into second-class factor F_{ij} ($i = 1,2,\cdots,n$; $j = 1,2,\cdots,m_i$), where m_i is the number of evaluating element for the i main characteristic factor. The number of second-class factor can be different, namely different first-class factor F_i can be made up of different number m_i of second-class factor.

The second-class factor may be made of the next class factor.

The evaluating criterion set of every factor is

$H = \{H_{ijl}\}(i = 1,2,\cdots,n\ ,\ j = 1,2,\cdots,n_i, l = 1,2,\cdots,p)$.

The evaluating grade set is

$V = \{V_l\}(l = 1,2,\cdots,p)$.

According to the evaluating criterion of industrial design knowledge, the mark grade of every evaluating factor is set down. Thus the qualitative evaluating factor of industrial design will be transformed into a quantitative set. It is separately evaluated with the fuzzy ratiocination and the grey ratiocination separately. And then they are composed with the evidential reasoning approach. Meanwhile, the estimator and expert judge the comparative essentiality of evaluating factor and criterion of a kind of product with administrative levels analyzing and comparative method. Based on it, the weight vector quantity of the evaluating factor is confirmed and weight matrixes are constructed as

$W = \{W_1, W_2, \cdots, W_n\}, W_i = \{W_{i1}, W_{i2}, \cdots, W_{im_{ii}}\}\ \cdots$

Where W_i is the corresponding weight of F_i, and $\sum_{i=1}^{n} W_i = 1$, $0 < W_i < 1$. W_{ij} is the corresponding weight of F_{ij}, and $\sum_{j=1}^{m_i} W_{ij} = 1$ □$0 < W_{ij} < 1$. According to the evaluating factor set, the weight matrix of the next one can be constructed.

Supposed that there are a estimators and its serial number is k, so estimator set is $E = \{E_1, E_2, \cdots, E_a\}$. According to his or her knowledge level, the weight matrix of expert is confirmed as $A = \{A_1, A_2, \cdots, A_a\}$.

2.2 Evaluation based on the grey theory

2.2.1 Constructing the sample matrix of grey evaluation

Based on the evaluating criterion, the estimators give a mark of every factor and fill in the evaluating form. According to the mark $d_{ijk}^{(s)}$ given by the k estimator for the s sample, the sample matrix of grey evaluation is contracted as

$$D^{(s)} = (d_{ijk}^{(s)})_{n\times(m_1+m_2+\cdots+m_n)\times a}\ .$$

Where $i = 1,2,\cdots,n$; $j = 1,2,\cdots,m_i; k = 1,2,\cdots,a$.

2.2.2 Confirming the evaluation grey classes

According to evaluating requirement of industry design, grey evaluation classes can be divided into p classes. The serial number of grey evaluation classes is l and the corresponding grey value is $\otimes = (\otimes_1,\otimes_2,\cdots,\otimes_p)$. Through the industry design evaluation criterion and expertise are analyzed, the whiting weight function $f_l(d_{ijk}^{(s)})$ can be confirmed respectively. The threshold value is d_l $(l = 1,2,\cdots,p)$. The threshold value of top part is d_0 and the threshold value of under part is $d_{(p+1)}$ respectively. Its general formula is expressed as:

for top part value ($l = 1$),

$$f_1(d_{ijk}^{(s)}) = \begin{cases} (d_{ijk}^{(s)} - d_0)/(d_1 - d_0), & d_{ijk}^{(s)} \in [d_0, d_1] \\ 1 & , d_{ijk}^{(s)} \in [d_1, d_2] \\ 0 & , d_{ijk}^{(s)} \notin [d_0, d_2] \end{cases};$$

for midst part value ($l = 2,3,\cdots,(p-1)$),

$$f_l(d_{ijk}^{(s)}) = \begin{cases} (d_{ijk}^{(s)} - d_{(l-1)})/(d_l - d_{(l-1)}), & d_{ijk}^{(s)} \in [d_{(l-1)}, d_l] \\ (d_{(l+1)} - d_{ijk}^{(s)})/(d_{(l+1)} - d_l), & d_{ijk}^{(s)} \in [d_l, d_{(l+1)}] \\ 0 & , d_{ijk}^{(s)} \notin [d_{(l-1)}, d_{(l+1)}] \end{cases};$$

for under part value ($l = p$),

$$f_p(d_{ijk}^{(s)}) = \begin{cases} 1 & , d_{ijk}^{(s)} \in [d_{(p-1)}, d_p] \\ (d_{(p+1)} - d_{ijk}^{(s)})/(d_{(p+1)} - d_p), & d_{ijk}^{(s)} \in [d_p, d_{(p+1)}] \\ 0 & , d_{ijk}^{(s)} \notin [d_{(p-1)}, d_{(p+1)}] \end{cases}.$$

2.2.3 Calculating the grey evaluation value

For evaluation factor F_{ij} of the s sample that belongs to the l grey evaluation class is written as $x_{ijl}^{(s)}$. Considering each expert's weight, it is formulated as

$$x_{ijl}^{(s)} = \sum_{k=1}^{a} A_k f_l(d_{ijk}^{(s)}).$$

The s evaluation sample that belongs to the whole grey evaluation is formulated as

$$x_{ij}^{(s)} = \sum_{l=1}^{p} x_{ijl}^{(s)}\ .$$

2.2.4 Calculating the grey evaluation vector

For the evaluation index F_{ij} of the s sample, the value that all estimators are evaluating in the l grey class is titled as the grey evaluation vector. It is formulated as

$$g_{ijl}^{(s)} = \frac{x_{ijl}^{(s)}}{x_{ij}^{(s)}}.$$

Then, it is written as

$G_i^{(s)} = (g_{ijl}^{(s)})$.

2.3 Evaluation base on the fuzzy theory

2.3.1 Constructing the sample matrix of fuzzy evaluation

Each item is divided into p grades that are according with grey evaluation classes. Based on the evaluating criterion, the estimators give a choice of every factor and fill in the evaluating grade form. It is expressed as $t_{ijkl}^{(s)}$. Where s is the serial number of sample; k is the serial number of estimator; l is the serial number of evaluating grade. If the l grade is selected, $t_{ijkl}^{(s)}$ is 1. Or else $t_{ijkl}^{(s)}$ is 0. Then the sample matrix of fuzzy evaluation is contracted as

$$T^{(s)} = (t_{ijkl}^{(s)})_{n \times (m_1 + m_2 + \cdots m_n) \times a \times p} .$$

Where $i = 1,2,\cdots,n; j = 1,2,\cdots,m_i; k = 1,2,\cdots,a; l = 1,2,\cdots,p$.

2.3.2 Calculating the fuzzy evaluation value

For evaluation factor F_{ij} of the s sample that belongs to the l fuzzy evaluation grade is written as $z_{ijl}^{(s)}$. Considering each expert's weight, it is formulated as

$$z_{ijl}^{(s)} = \sum_{k=1}^{a} A_k t_{ijkl}^{(s)} .$$

The s evaluation sample that belongs to the whole fuzzy evaluation is formulated as

$$z_{ij}^{(s)} = \sum_{l=1}^{p} z_{ijl}^{(s)} .$$

2.3.3 Calculating the fuzzy evaluation value

For the evaluation index F_{ij} of the s sample, the value that all estimators are evaluating in the l fuzzy class is titled as the fuzzy evaluation vector. It is formulated as

$$y_{ijl}^{(s)} = \frac{z_{ijl}^{(s)}}{z_{ij}^{(s)}} .$$

Then, it is written as

$$Y_i^{(s)} = (y_{ijl}^{(s)}) .$$

2.4 Composing evaluation data with the evidential reasoning approach [6][7]

The evidential reasoning approach is very efficient for dealing with the uncertain information in multiple-attribute hierarchical evaluation process. With the evidential reasoning approach, the grey evaluation data and the fuzzy evaluation data are combined. Its logical algorithm is

$$r_{ijl}^{(s)} = M_{ij}^{(s)} (g_{ijl}^{(s)} y_{ijl}^{(s)}) .$$

Where $M_{ij}^{(s)} = [1 - \sum_{l'=1}^{p} \sum_{l=1, l \neq l'}^{p} g_{ijl'}^{(s)} y_{ijl}^{(s)}]^{-1}$.

Then the composing result is contracted as

$$R_i^{(s)} = \begin{bmatrix} r_{i1}^{(s)} \\ r_{i2}^{(s)} \\ \vdots \\ r_{im_i}^{(s)} \end{bmatrix} = \begin{bmatrix} r_{i11}^{(s)} & r_{i12}^{(s)} & \cdots & r_{i1p}^{(s)} \\ r_{i21}^{(s)} & r_{i22}^{(s)} & \cdots & r_{i2p}^{(s)} \\ & \vdots & & \\ r_{im_i1}^{(s)} & r_{im_i2}^{(s)} & \cdots & r_{im_ip}^{(s)} \end{bmatrix}.$$

2.5 Comprehensive evaluation

2.5.1 Comprehensive evaluation

For comprehensive evaluation on F_i of the s evaluation sample, the result is written as $B_i^{(s)}$. It is formulated as

$$B_i^{(s)} = W_i \bullet R_i^{(s)} = (b_{i1}^{(s)}, b_{i2}^{(s)}, \cdots, b_{ip}^{(s)}).$$

All $B_i^{(s)}$ are composing of $R^{(s)}$. It is written as

$$R^{(s)} = (B_1^{(s)}, B_2^{(s)}, \cdots, B_p^{(s)})^T.$$

The next comprehensive evaluating result is formulated as

$$B^{(s)} = W \bullet R^{(s)} = (b_1^{(s)}, b_2^{(s)}, \cdots, b_p^{(s)}).$$

2.5.2 Calculating the comprehensive evaluation value and arranging the result

Comprehensive evaluation result $B^{(s)}$ of evaluation sample s is a vector. It will lose some information if the maximum value is selected. Therefore the vector should be simplified.

Each evaluation grade is evaluated with according to threshold value of grey evaluation. The vector value of evaluation grade is written as

$$C = \{d_l\} \quad (l = 1,2,\cdots,p).$$

The last comprehensive evaluation value is formulated as

$$F^{(s)} = B^{(s)} \bullet C^{\mathrm{T}}.$$

$F^{(s)}$ is the comprehensive evaluation value of the industry design evaluation to the evaluation sample s. Based on $F^{(s)}$, all evaluation samples will be arranged.

3. Applying example and validation test

The evaluation samples are machine tools. There are 4 style samples and 8 estimators. This paper analyses the single factor that is the ergonomic evaluation. The evaluation criterion is shown as Fig 1.

3.1 Establishing the grade of the evaluation criterion

The every evaluation factor is divided into 4 grades and every evaluation grade criterion is made according to different levels.

3.2 Confirming the weight of the evaluation factor

The weight vector of the evaluation factor is

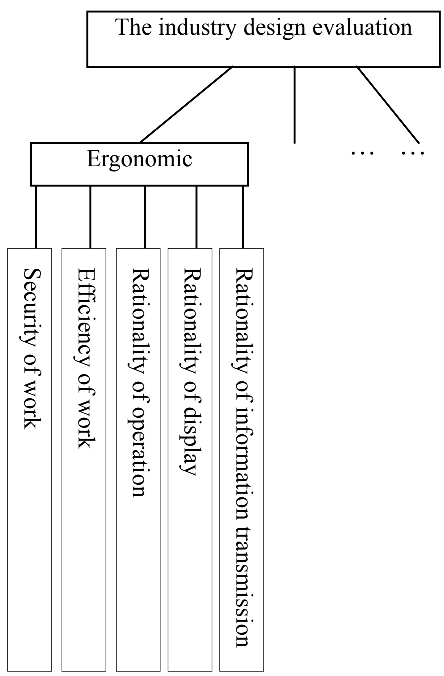

Fig1. Evaluation structure chart

W_1={0.196,0.187,0.193,0.185,0.239}.

Every expert's knowledge has the same level that the weight value is 0.8.

3.3 Evaluation based on the grey theory

3.3.1 Establishing grey evaluation sample matrix

According to the grade form that is filled in by the 8 estimators, the grey evaluation sample matrix $D_1^{(1)}$ of ergonomic evaluation of the 1st machine tool is the following tab 1.

With the same method, the 2nd, 3rd and 4th evaluating samples matrix for machine tools, $D_1^{(2)}, D_1^{(3)}$ and $D_1^{(4)}$, are established.

Tab 1. Grey evaluation data of ergonomic evaluation of the 1st machine tool

	F_{11}	**F_{12}**	**F_{13}**	**F_{14}**	**F_{15}**		**F_{11}**	**F_{12}**	**F_{13}**	**F_{14}**	**F_{15}**
E_1	9	8	9	8	8	**E_5**	9	8	8	8	8
E_2	8	7	8	9	9	**E_6**	8	6	7	9	9
E_3	8	7	10	8	9	**E_7**	8	6	8	8	7
E_4	7	6	8	7	8	**E_8**	8	7	7	8	8

3.3.2 Confirming grey evaluation class

According to evaluating requirement, grey evaluation classes are divided into 4 classes. Its corresponding threshold value is 9, 7, 5 and 2. And then the right whiting value functions are constructed.

3.3.3 Calculating the grey evaluation vector

With calculating step by step, the grey evaluation vector of the evaluation index F_{1j} is written as

$$G_1^{(1)} = \begin{bmatrix} 0.5625 & 0.4375 & 0 & 0 \\ 0.125 & 0.6875 & 0.1875 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.5625 & 0.4375 & 0 & 0 \\ 0.625 & 0.375 & 0 & 0 \end{bmatrix}.$$

3.4 Evaluation based on the fuzzy theory

3.4.1 Establishing fuzzy evaluation sample matrix

According to grey evaluation classes, the evaluation grades are dividing into four grades that are "A", "B ", "C" and "D". Its corresponding quantity value is 9, 7, 5 and 2. Then its fuzzy evaluation sample is as follows.

Tab 2. Fuzzy evaluation data of ergonomic evaluation of the 1st machine tool

	t_{11}				**t_{12}**				**t_{13}**				**t_{14}**				**t_{15}**			
	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D	A	B	C	D
E_1	1	0	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0
E_2	0	1	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0
E_3	0	1	0	0	0	1	0	0	1	0	0	0	0	1	0	0	1	0	0	0
E_4	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0
E_5	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0	1	0	0	0
E_6	0	1	0	0	0	0	1	0	0	1	0	0	1	0	0	0	1	0	0	0
E_7	0	1	0	0	0	0	1	0	0	1	0	0	0	1	0	0	0	1	0	0
E_8	1	0	0	0	0	1	0	0	0	1	0	0	1	0	0	0	1	0	0	0

3.4.2 Calculating e fuzzy evaluation vector

With calculating step by step, the fuzzy evaluation vector of the evaluation index F_{1j} is written as

$$Y_1^{(1)} = \begin{bmatrix} 0.375 & 0.625 & 0 & 0 \\ 0.125 & 0.5 & 0.375 & 0 \\ 0.375 & 0.625 & 0 & 0 \\ 0.5 & 0.5 & 0 & 0 \\ 0.75 & 0.25 & 0 & 0 \end{bmatrix}.$$

3.5 Composing evaluation data with the evidential reasoning approach

With the evidential reasoning approach, the grey evaluation data and the fuzzy evaluation data are combined as

$$R_1^{(1)} = \begin{bmatrix} 0.4355 & 0.5645 & 0 & 0 \\ 0.0364 & 0.8 & 0.1636 & 0 \\ 0.375 & 0.625 & 0 & 0 \\ 0.5625 & 0.4375 & 0 & 0 \\ 0.8333 & 0.1667 & 0 & 0 \end{bmatrix}.$$

3.6 Comprehensive evaluation of F_1

The result comprehensive evaluation of F_1 of the 1st evaluating machine tool is formulated as

$$B_1^{(1)} = W_1 \bullet R_1^{(1)} = (0.4678 \quad 05016 \quad 0.0306 \quad 0).$$

Then,

$$F_1^{(1)} = B_1^{(1)} \bullet C^{\mathrm{T}} = [0.4678 \quad 05016 \quad 0.0306 \quad 0]*[9 \quad 7 \quad 5 \quad 2]^{\mathrm{T}}$$

$$=7.8744.$$

The comprehensive evaluation value of other design is gained with the same method. Therefore, a rational design project can be finally elected based on arranging of evaluation result.

3.7 Validation test

The validation of the proposed algorithm was tested by means of comparing the difference of the evaluation results based on the proposed algorithm and the evaluation results based on the previous methods – the single fuzzy theory and the single grey theory. Table 3 shows the results.

Tab.3 The results of validation test based on the different methods

	Group1	Group2	Group3	Group4	Group5	Group6	Std. Deviation
Method 1	8.4345	8.2340	8.3363	8.3226	8.3926	8.3649	0.0686
Method 2	8.7731	8.5834	8.2418	8.7094	8.8729	8.4828	0.2272
Method 3	8.0360	7.8659	8.1355	7.8023	7.7940	8.2651	0.1938

A machine tool is selected to be the evaluation sample and all evaluation indexes of industrial design are analyzed. There are 6 groups of estimators that each group includes 8 estimators. The method 1 is the evaluation method based on the fuzzy theory and grey theory; the method 2 is the evaluation method based on the grey theory; the method 3 is the evaluation method based on the fuzzy theory. The value of table is the evaluation result. The last column is the Std. deviation of different method. It can be said that there are differences between the evaluation precisions by the Std. deviation such as 0.0686<0.1938<0.2272. This table shows that the evaluation method based on the fuzzy theory and grey theory in this paper is provided with the highest precision. Therefore, it is the best method for evaluating subjective index of industrial design.

4. Conclusion

(1) Based on the fuzzy theory and grey theory, the evaluation of the industry design is carried on. And then the evaluating data is composed with the evidential reasoning approach. This algorithm is fully utilizing of evaluation data of every factor to infer the overall situation objectively. Therefore, this algorithm can give overall and rational industrial design synthesis evaluation according to various kinds of situation.

(2) With this algorithm, it had realized the computer assists industry design evaluate system. Through the network, it will fully utilize anywhere expert resources and will effectively help to industrial design. So, this algorithm has important reference value on the data processing in constructing the intelligent industry design evaluation system in the future.

Acknowledgment

The author is grateful to the excellence younger teacher fund of Lanzhou University of Science & Technology for supporting this research.

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