Total Absolute Curvature to Represent The Complexity of Diverse Curved Profiles

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Abstract: In curve design, controlling the macroscopic feature that emerges from the combinations of shape elements is important. However, controlling the macroscopic feature is difficult using conventional microscopic shape-information such as dimension and curvature. Therefore, in curve design, a design-support system that uses macroscopic shape-information to represent the macroscopic feature is desired. In the our past study, the information entropy calculated by the discrete curvature function as Markov Process was found to represent the complexity of basic curved profiles described by three-dimensional Bézier curve. Besides, it was showed the effectiveness of the information entropy as a shape-generation index. However, the existence of the curved profiles that the information entropy is difficult to apply was pointed out. The objective of the present study is to show that the total absolute curvature as macroscopic shape-information that represents the complexity of diverse curved profiles. Firstly, the total absolute curvature was applied to the basic curved profiles. The relationship between the total absolute curvature and complexity was analyzed. As a result, the total absolute curvature was found to represent the complexity as well as the information entropy. Secondly, the information entropy and the total absolute curvature were applied to the curved profiles of automobile side-views described by three-dimensional Bézier curve. As a result, the information entropy was found to be difficult to apply. On the other hand, the total absolute curvature was found to represent the complexity. In the present study, it was showed that the total absolute curvature represents the complexity of diverse curved profiles comparing with the information entropy.
***Key words*:** *Curve Design, Macroscopic Shape-Information, Macroscopic Feature, Total Absolute Curvature*

1. Introduction

In curve design, a designer mainly connects curved profiles using smoothness as an index. Smooth curved profiles are advantage to molding and manufacturing, but are restricted to consider new shapes. In considering curved profiles, human beings tend to perceive overall shape feature macroscopically [1-5]. Then, the control of the macroscopic feature that emerges from the combinations of shape elements is important. In CAD (Computer Aided Design), however, the microscopic shape-information such as dimension and curvature is presented, but the macroscopic shape-information is not presented. Therefore, in curve design, a design-support system that uses the macroscopic shape-information to represent the macroscopic feature is desired.

In our past study, the information entropy calculated by the curvature function of curved profiles was proposed. The relationship between the information entropy and the macroscopic feature "complexity" was analyzed on the basic curved profiles and the generated shapes. The macroscopic feature "complexity" is indicated to be one of the factors of "beauty" [6]. As a result, the information entropy calculated by the discrete curvature function as the Markov process was to be a new shape-information to represent the complexity [7-10]. However, the existence of the curved profiles that the information entropy is difficult to apply was pointed out.

In the present study, firstly, the new shape-information was proposed. Secondly, the proposed shape-information and the information entropy are applied to the new generated basic curved profiles. Finally, both macroscopic shape-information is applied to the curved profiles generated from the products. The objective of this study is to show that the proposed new macroscopic shape-information represents the complexity of diverse curved profiles.

2. Information Entropy

In the study by Jernigan [11] and Thiran [12], the complexity of pattern was quantified by the information entropy. The information entropy is the value represent to the dispersion of events. This value needs to be discrete by sampling and quantization because of targeting not to continuous quantity but to discrete quantity. In our past study, the information entropy calculated by the discrete curvature function as the Markov process was proposed as new shape-information to represent the complexity [7-10].

The information entropy is calculated from curvature function of the curved profile in the following manner. In Fig.1, the vertical axis is curvature κ, the horizontal axis is the curve length l, the curvature function is $\kappa(l)$ and the total length of curved profile is L. First, curved profile was divided by arbitrary number N (N=12 in Fig.1) and curvature in each point was calculated (sampling). Next, the range of the curvature function is divided by arbitrary number V (V=3 in Fig.1). A series of symbols was fitted in a curvature of each range (quantization). Then, a series of symbols consisting of S_i, which are V kinds, is modeled by Markov process. Finally, q_i (the occurrence probability of S_i) and $q_{i,j}$ (the transition probability of S_i to S_j) were calculated. The information entropy H was calculated using the following equation:

$$H = -\frac{1}{\log_2 V} \sum_{i=1}^{V} \sum_{j=1}^{V} q_i q_{i,j} \log_2 q_{i,j} \quad 0 \le H \le 1 \tag{1}$$

The number N, V, and the range of the curvature function $\kappa(l)$ are the parameters to be set. In our past study, the combination to which the correlation coefficient of the information entropy and the complexity in basic curved profiles becomes the highest was selected from arbitrary setup of each parameter. Therefore, the existence of the curved profiles that the information entropy is difficult to apply was pointed out.

3. Total Absolute Curvature

In the knowledge of the study about the complexity in outline shapes, the number of vertices is cited as one of factors of the complexity [13, 14]. A vertex is the feature point for describing straight-line profile. The feature point in the curved profile is equivalent to a high curvature point. Therefore, it is considered that the number of high curvature points cause "complexity" in the curved profile. However, in order that curvature changes continuously, the criterion that divides a high curvature point and the other point is needed as a parameter. So, in the present study, the number of high curvature points was not computed by having set up the parameter, but the total of the absolute curvature was computed as index corresponding to that. This value is known as the total absolute curvature that is one of the global properties of curved profile in the differential geometry [15, 16].

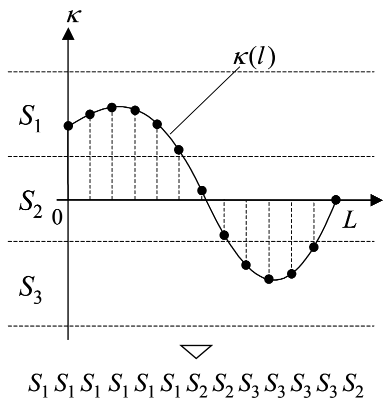

Fig.1 Extraction of Information Entropy Based on Curvature Function

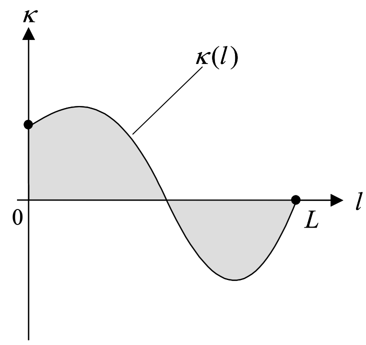

Fig.2 Extraction of Total Absolute Curvature Based on Continuous Curvature Function

The total absolute curvature from curvature function in the curved profile is calculated in the following manner. In Fig.2, the vertical axis is curvature κ, the horizontal axis is the curve length l, the curvature function is $\kappa(l)$ and the total length of curved profile is L. The total absolute curvature is calculated using the following equation:

$$T = \frac{1}{2\pi}\int_0^L |\kappa(l)|dl \qquad (2)$$

In this equation, the total absolute curvature is made the dimensionless value divided by 2π. In closed curved profile, the minimum of total absolute curvature is 1, and in open curved profile, the minimum of that is 0.

4. Application to The Basic Curved Profiles

In this chapter, the relationship between the macroscopic shape-information and the complexity is analyzed in some basic curved profiles described by three-dimensional Bézier curve.

In order to describe the basic curved profiles, 20 polygonal profiles were selected. In this regard, the polygonal profiles were used in "Aesthetic Measure" by Birkhoff [6], who is the pioneer in the field of the study of "aesthetic" through the experimental psychology approach. The basic curved profiles were described by setting junction points and control points for these polygonal profiles, and interpolated by three-dimensional Bézier curve under C^1 continuity. All of the described basic curved profiles are shown in Fig.3.

In the present study, the cognition experiment was carried out to obtain the quantitative value of the complexity in the basic curved profiles under the following conditions.

Method	:	semantic differential method (5 stages)
Evaluation items	:	"complex"
Samples	:	basic curved profiles (20 samples)
Examinees	:	13 subjects in 20 generations

The distance between subject and CRT (Cathode Ray Tube) display was set 600mm referring to the knowledge that the best distance between the operator and CRT display is from 450 mm to 700 mm in VDT (Visual Display Terminal).

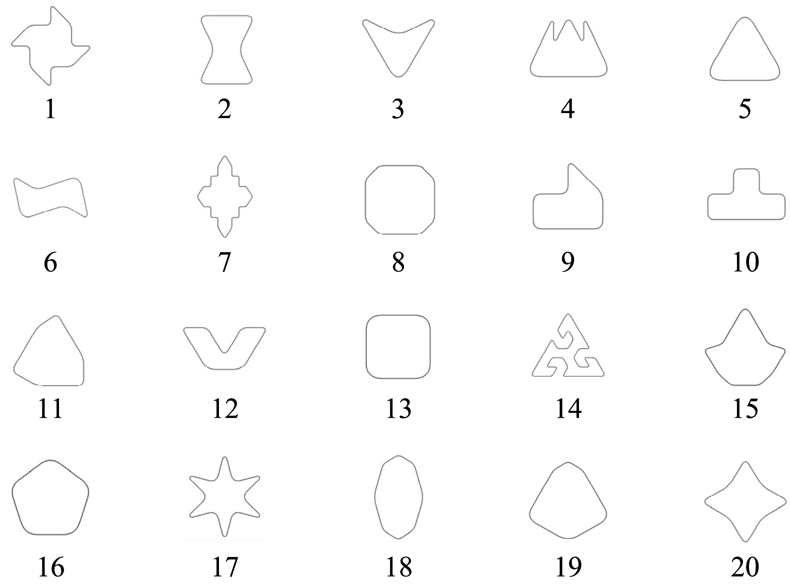

Fig.3 Basic Curved Profiles

The Relationship between the macroscopic shape-information (the information entropy and the total absolute curvature) computed from the basic curved profiles and the evaluation value of complexity was analyzed. As a result, the logarithmic function of the macroscopic shape-information was found to represent the complexity of basic curved profiles (as shown in Fig.4, 5).

5. Application to The Automobile Side-Views

In this chapter, the relationship between the macroscopic shape-information and the complexity is analyzed in some automobile side-views described by three-dimensional Bézier curve. In the present study, the curved profiles extracted from product shape are used.

In order to describe the automobile side-views, some automobiles were selected. The various automobile side-views of three types, which are Coupe, SUV (Sports Utility Vehicle) and Sedan, were provided from TOYOTA co. 6 automobile side-views were selected from each type.

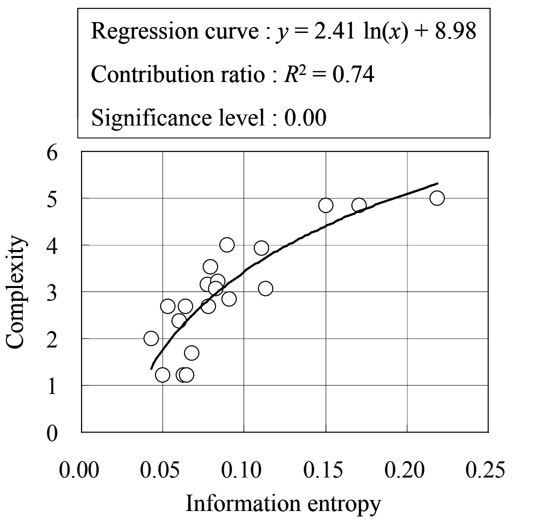

Fig.4 Relationship between Information Entropy and Complexity in Basic Curved Profiles

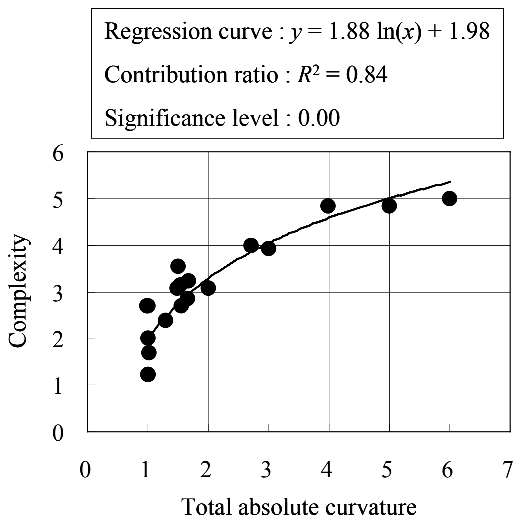

Fig.5 Relationship between Total Absolute Curvature and Complexity in Basic Curved Profiles

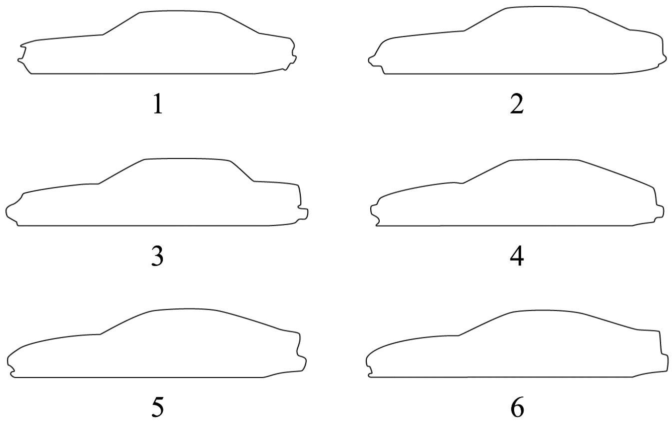

Fig.6 Automobile Side-Views (Coupe)

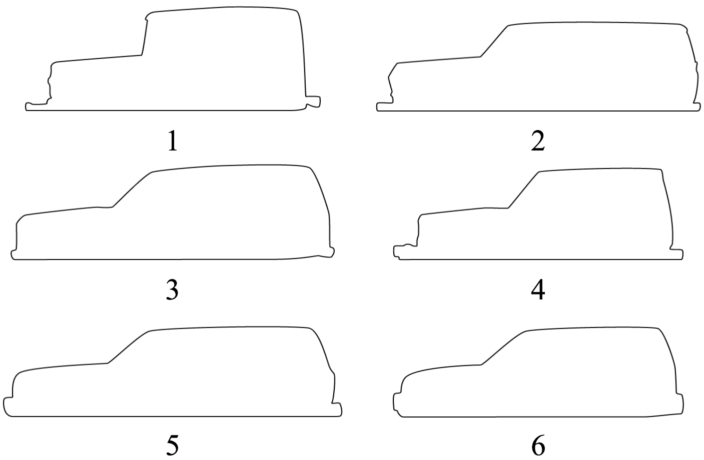

Fig.7 Automobile Side-Views (SUV)

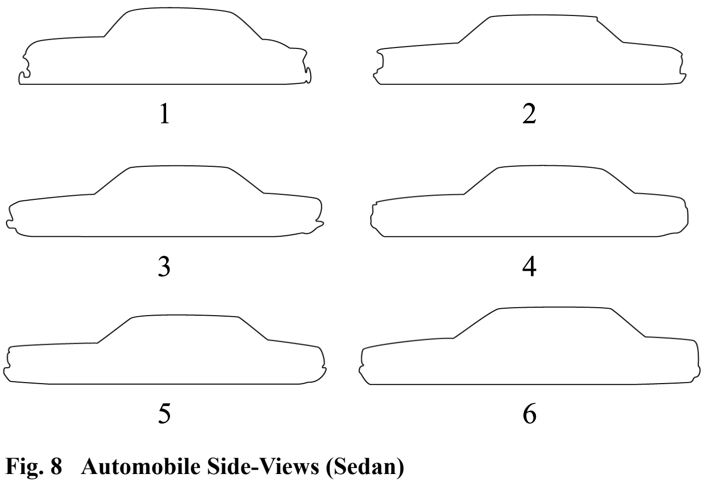

Fig. 8 Automobile Side-Views (Sedan)

The outlines of the automobile side-views were described by the three-dimensional Bézier curve under C^1 continuity. The drawings of the automobile side-views were scanned as image, and noise removal was carried out using Photoshop 5.5 of Adobe. Next, the outlines of the automobile side-views were extracted as three-dimensional Bézier curve using Illustrator 9.0 of Adobe. Then, the extracted three-dimensional Bézier curves were corrected considering the low dispersion of curvature in each segment and the C^1 continuity in each junction point. All of the described automobile side-views are shown in Fig.6, Fig.7, and Fig.8.

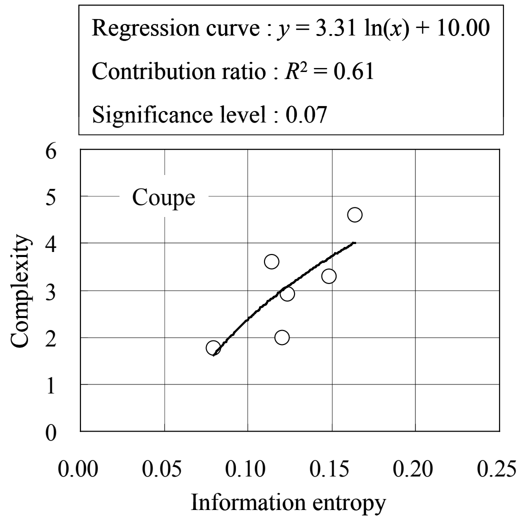

Fig. 9-a Relationship between Information Entropy and Complexity in Automobile Side-Views (Coupe)

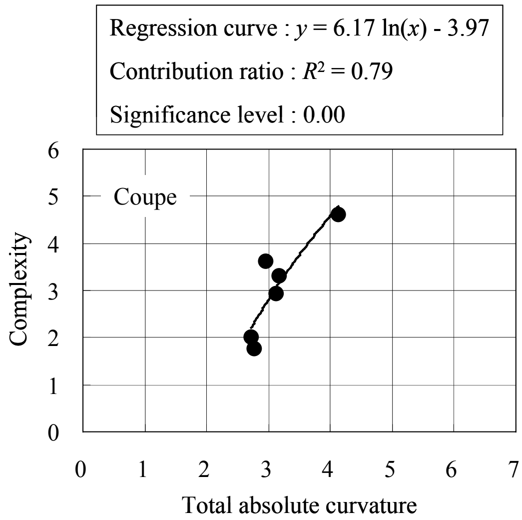

Fig. 10-a Relationship between Total Absolute Curvature and Complexity in Automobile Side-Views (Coupe)

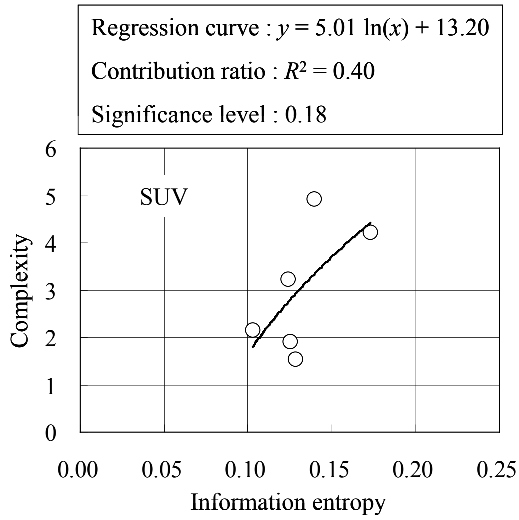

Fig. 9-b Relationship between Information Entropy and Complexity in Automobile Side-Views (SUV)

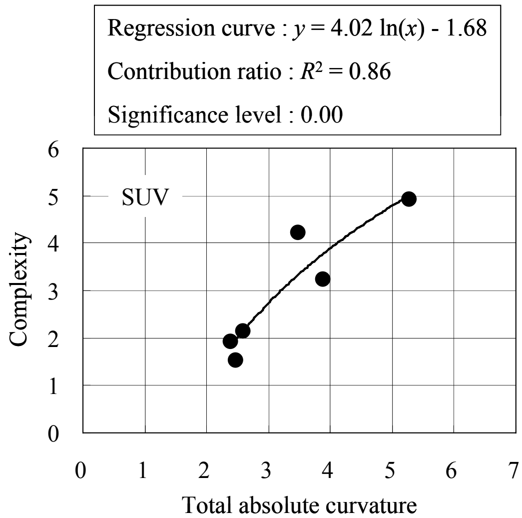

Fig. 10-b Relationship between Total Absolute Curvature and Complexity in Automobile Side-Views (SUV)

In the present study, the cognition experiment was carried out to obtain the quantitative value of the complexity in the automobile side-views under above mentioned conditions in chapter **4**. The distance between subject and CRT display was set 600mm like the above mentioned.

The Relationship between the macroscopic shape-information (the information entropy and the total absolute curvature) computed from the automobile side-views and the evaluation value of complexity was analyzed. As a result, the logarithmic function of the information entropy was found to be difficult to represent the complexity of the automobile side-views (as shown in Fig.9). On the other hand, the logarithmic function of the total absolute curvature was found to represent the complexity of the automobile side-views (as shown in Fig.10).

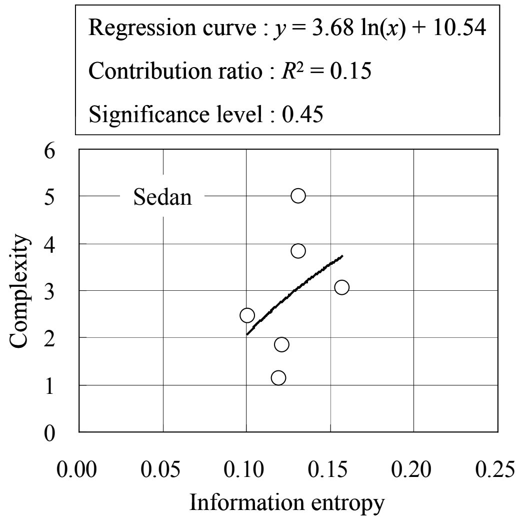

Fig. 9-c Relationship between Information Entropy and Complexity in Automobile Side-Views (Sedan)

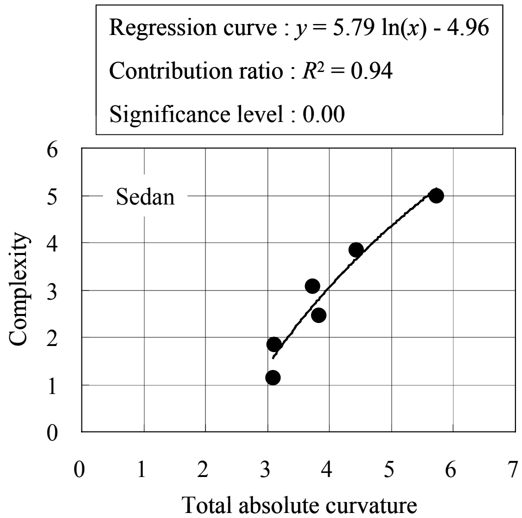

Fig. 10-c Relationship between Total Absolute Curvature and Complexity in Automobile Side-Views (Sedan)

6. Conclusion

In the present study, the total absolute curvature was proposed as new macroscopic shape-information to represent the complexity of curved profiles. Then, the relationship between the total absolute curvature and the complexity was analyzed on the basic curved profiles and the automobile side-views. As a result, the logarithmic function of the total absolute curvature was highly correlated with both curved profiles. Thus, the possibility of that the total absolute curvature represents the complexity of diverse curved profiles was indicated.

From this result, human beings tend to perceive macroscopic feature "complexity" paying attention to the number of high curvature points. However, on the basic curved profiles, the possibility of that the information entropy represents the complexity was showed. By setting up a parameter appropriately, it can represent the complexity of the automobile side-views. Therefore, the possibility of that human beings tend to perceive macroscopic feature "complexity" paying attention to the disorder of curvature change cannot be denied only by the result of the present study. About these subjects, it is necessary to perform detailed verification from a viewpoint of cognitive science.

Moreover, in the future, research on application to curve design such as method for shape-generation using the total absolute curvature and genetic algorithm is advanced.

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