Array Calibration for Compensating Gain/Phase Mismatch and Mutual Coupling Effects in Smart Antenna Systems

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***Abstract* - In smart antenna downlink systems, the beamforming is severely affected by antenna gain and phase mismatch as well as mutual coupling effects. We propose an eigen-structure based approach to estimate the gain and phase of each antenna sensor and mutual coupling coefficients between antenna sensors. By preprocessing the transmission signal in base station with the estimated parameters, the gain and phase mismatch and mutual coupling effects are compensated. Simulation results with data collected from a smart antenna test bed show that the estimation error is less than 5% and correct beam pattern is obtained.**

I. INTRODUCTION

Smart Antennas is one of the key techniques for wireless communications [1]. In Smart Antenna Systems (SAS), antenna array is used to transmit and receive signal directionally so that the interference is refrained and the system capacity is enhanced [2]. But in many real situations, the performance of the antenna array is degraded due to the gain and phase imbalance between antenna sensors, unknown mutual coupling coefficients, unknown sensor location, and so on. Therefore, antenna array calibration is indispensable.

Antenna array calibration mainly compensates the following factors: 1) the gain and phase mismatch due to unknown sensor location, 2) the gain and phase mismatch between antenna sensors, and 3) mutual coupling between antenna sensors. In this paper, we focus on the latter two factors.

Several methods for antenna array calibration have been proposed. The first type of methods is to use signals from known sources and known directions for the calibration purpose. B. Friedlander and A. J. Weiss [3] presented a method to estimate gain and phase of each sensor and mutual coupling in the receiving array for smart antenna uplink systems. The second type of method is to inject an equal phase signal to all channel of the antenna array. R. B. Ertel et al. [4] reported the technique to compensate gain and phase imbalance in uplink system of antenna array. The third type of method is to use network analyzer for calibration. K. R. Dandekar et al. [5] gave a mathematical model of smart antenna and calibrated the smart antenna array by network analyzer measurements and computational electromagnetic (CEM) simulations. Other techniques include uplink calibration by antenna switching system [6] and calibration procedure by utilizing an additional antenna for phase compensation [7].

In this paper, we propose an eigen-structure based approach for smart antenna downlink system calibration to guarantee the performance of transmission. The compensation procedure covers gain and phase mismatch as well as mutual coupling effects between antenna sensors. The good performance of downlink beamforming is shown by simulation results with data collected from a smart antenna test bed in Xi'an Jiaotong University.

The paper is organized as follows. Section II explains signal model and calibration principle. Section III describes the estimation procedure and algorithm. Section IV shows simulation results. Section V concludes the paper.

II. SIGNAL MODEL AND CALIBRATION PRINCIPLE

A. Signal Model

Consider the antenna array has M sensors, the beamforming direction from base station is θ_b, the mobile terminal is located at θ_m and the mobile terminal has one element.

Under the ideal condition without gain / phase mismatch and mutual coupling effects of antenna sensors, the received signal of mobile terminal is given by

$$x(t) = \mathbf{a}^H(\theta_m)\mathbf{a}(\theta_b)s(t) + n(t) \qquad (1)$$

where $s(t)$ is the transmitted signal, $\mathbf{a}(\theta_m)$ and $\mathbf{a}(\theta_b)$ are $M \times 1$ steering vectors for direction θ_m and θ_b, $n(t)$ is noise, $[\bullet]^H$ represents the conjugate transpose operation.

In practice, the gain / phase mismatch and mutual coupling effects of antenna sensors should be included in the signal model. Then, the received signal of mobile terminal can be expressed as

$$r(t) = \mathbf{a}^H(\theta_m)\mathbf{Z}\mathbf{a}(\theta_b)s(t) + n(t) \quad (2)$$

$$\mathbf{Z} = \mathbf{C}\boldsymbol{\Gamma} \quad (3)$$

where $\mathbf{Z}$ is an $M \times M$ complex matrix representing gain/phase mismatch and mutual coupling effects of antenna sensors, $\mathbf{C}$ is an $M \times M$ complex matrix representing mutual coupling, Γ is an $M \times M$ complex diagonal matrix whose elements $\left\{g_m e^{j\phi_m}\right\}_{m=1}^{M}$ represent the sensor gains and phases [8].

Model of the mutual coupling matrix $\mathbf{C}$ was presented in [3]. For a linear uniform array, $\mathbf{C}$ is a Toeplitz matrix. The rationale behind this model is the fact that the mutual coupling coefficients are inversely proportional to the distance between the elements. For a uniform circular array $\mathbf{C}$ is a circulant matrix.

B. Problem Description

To compensate the sensor gain / phase mismatch and mutual coupling effects, we need to obtain $\hat{\mathbf{Z}}$ (the estimation of $\mathbf{Z}$) and use it to preprocess the transmitting signal in base station. Fig.1 illustrates the conceptual diagram of array calibration for downlink smart antenna systems. With the concept of software radio, beamforming can be achieved by applying compensated weighing vector in the base station.

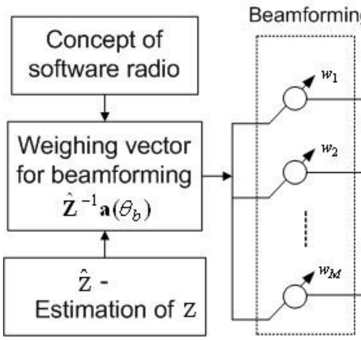

Fig. 1 Conceptual diagram of array calibration for downlink smart antenna systems.

The compensated weighing vector is given by

$$\tilde{\mathbf{a}}(\theta_b) = \hat{\mathbf{Z}}^{-1}\mathbf{a}(\theta_b) \quad (4)$$

where $\hat{\mathbf{Z}}^{-1}$ is the $M \times M$ inverse matrix of $\hat{\mathbf{Z}}$, $\tilde{\mathbf{a}}(\theta_b)$ is an $M \times 1$ complex vector.

Use $\tilde{\mathbf{a}}(\theta_b)$ as the downlink beamforming vector, that is, substitute $\tilde{\mathbf{a}}(\theta_b)$ for $\mathbf{a}(\theta_b)$ in (2), we obtain

$$r'(t) = \mathbf{a}^H(\theta_m)\mathbf{Z}\hat{\mathbf{Z}}^{-1}\mathbf{a}(\theta_b)s(t) + n(t) . \quad (5)$$

Compare Eq. (5) with Eq. (1), if the estimation of $\mathbf{Z}$ is accurate, $r'(t)$ will be the same as the ideal received signal by mobile terminal.

III. Estimation Procedure and Algorithm

In this section, we propose a eigen-structure based calibration approach for smart antenna downlink systems. Two mobile terminals with fixed locations will be used for the calibration. The estimation procedure is shown in Fig. 2. Firstly, we will construct covariance matrix for each mobile terminal. Then eigen analysis will be performed. Finally, an iteration algorithm will be used to obtain the estimates of gain / phase and mutual coupling coefficients.

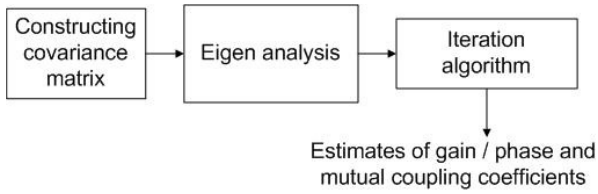

Fig. 2 Block diagram of the estimation procedure.

A. Obtaining the Covariance Matrix R

Assuming we have two mobile terminals located at θ_1 and θ_2 respectively, the covariance matrix for each mobile terminal is defined as

$$\mathbf{R}_n = \boldsymbol{\Gamma}^H \mathbf{C}^H \mathbf{a}(\theta_n)\mathbf{a}^H(\theta_n)\mathbf{C}\boldsymbol{\Gamma} , \quad n = 1,2 \quad (6)$$

Where $\mathbf{R}_n$ is an $M \times M$ Hermitian matrix.

Before we perform eigen analysis on the covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$, we show how to construct the covariance matrix.

In practice, after analogue to digital conversion (A/D), in the mobile terminal, the received signal can be described by

$$r(k) = \mathbf{a}^H(\theta_m)\mathbf{Z}\mathbf{a}(\theta_b)s(k) + n(k) . \quad (7)$$

It is known that a different code is assigned to each downlink channel. Then we discuss the condition when the base station transmits two channel signals (two different codes) to the same mobile terminal.

Assuming the mobile terminal is located at θ, the beamforming direction of channel 1 is β and the beamforming direction of channel 2 is γ.

From (7), the received signal of channel 1 after code match filtering is given by

$$r_1(k) = \mathbf{a}^H(\theta)\mathbf{Z}\mathbf{a}(\beta)s_1(k) + n_1(k) \quad (8)$$

where $r_1(k)$ is a complex number, $s_1(k)$ is the transmitting signal of channel 1 and the transmitting signal can be all ones used for calibration, $n_1(k)$ is noise.

The received signal of channel 2 after code match filtering is given by

$$r_2(k) = \mathbf{a}^H(\theta)\mathbf{Z}\mathbf{a}(\gamma)s_2(k) + n_2(k) . \quad (9)$$

Let $y = \frac{\mathrm{E}\left[r_1^H(k)r_2(k)\right]}{\mathrm{E}\left[s_1^H(k)s_2(k)\right]}$ represents the covariance between the received signal of channel 1 and channel 2, where $\mathrm{E}[\bullet]$ represents expectation operation. Ignoring the noise, we have

$$y = \mathbf{a}^H(\beta)\mathbf{Z}^H \mathrm{a}(\theta)\mathrm{a}^{\mathrm{H}}(\theta)\mathrm{Za}(\gamma) . \quad (10)$$

Now, we are ready to explain the procedure to obtain $\mathbf{R}_1$ and $\mathbf{R}_2$. The base station will transmit signal of two channels directionally to two mobile terminals at fixed locations θ_1 and θ_2. The covariance matrix $\mathbf{R}_1$ and $\mathbf{R}_2$ will be obtained by beam scanning from the base station, that is to say, changing the transmission directions of the two channels.

Next, we will explain the way to obtain $\mathbf{R}_1$ for the mobile terminal located at θ_1. $\mathbf{R}_2$ for the mobile terminal located at θ_2 can be obtained in the similar way.

1) Fixing β_1 (the beamforming direction of channel 1)

When the direction of channel 2 is γ_1, $y_{11} = \frac{E\left[r_1^H(k) r_2(k)\right]}{E\left[s_1^H(k) s_2(k)\right]}$, then

$$\mathbf{R}_1 = \mathbf{Z}^H \mathrm{a}(\theta_1)\mathrm{a}^{\mathrm{H}}(\theta_1)\mathrm{Z}. \quad (11)$$

From (10) and (11), we have

$$y_{11} = \mathbf{a}^H(\beta_1)\mathbf{R}_1\mathbf{a}(\gamma_1). \quad (12)$$

Then, we let the direction of channel 2 changes to γ_2, we can get $y_{12} = \mathbf{a}^H(\beta_1)\mathbf{R}_1\mathbf{a}(\gamma_2)$.

Until the beam of channel 2 scans to γ_K, $[y_{11}\ y_{12}\ \cdots\ y_{1K}]$ is given by

$$[y_{11}\ y_{12}\ \cdots\ y_{1K}] = \mathbf{a}^H(\beta_1)\mathbf{R}_1[\mathbf{a}(\gamma_1)\ \mathbf{a}(\gamma_2)\ \cdots\ \mathbf{a}(\gamma_K)]. \quad (13)$$

2) Let the direction of channel 1 changes to β_2

Repeat the steps in a), we can get

$$[y_{21}\ y_{22}\ \cdots\ y_{2K}] = \mathbf{a}^H(\beta_2)\mathbf{R}_1[\mathbf{a}(\gamma_1)\ \mathbf{a}(\gamma_2)\ \cdots\ \mathbf{a}(\gamma_K)].$$

Until the beam of channel 1 scans to β_P, we get

$$\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1K} \\ y_{21} & y_{22} & \cdots & y_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ y_{P1} & y_{P2} & \cdots & y_{PK} \end{bmatrix} = \begin{bmatrix} \mathbf{a}^H(\beta_1) \\ \mathbf{a}^H(\beta_2) \\ \vdots \\ \mathbf{a}^H(\beta_P) \end{bmatrix} \mathbf{R}_1[\mathbf{a}(\gamma_1)\ \mathbf{a}(\gamma_2)\ \cdots\ \mathbf{a}(\gamma_K)] \quad (14)$$

where $\begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1K} \\ y_{21} & y_{22} & \cdots & y_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ y_{P1} & y_{P2} & \cdots & y_{PK} \end{bmatrix}$ is a $P \times K$ complex matrix, $[\mathbf{a}(\gamma_1)\ \mathbf{a}(\gamma_2)\ \cdots\ \mathbf{a}(\gamma_K)]$ is a $M \times K$ complex matrix, and $\begin{bmatrix} \mathbf{a}^H(\beta_1) \\ \mathbf{a}^H(\beta_2) \\ \vdots \\ \mathbf{a}^H(\beta_P) \end{bmatrix}$ is a $P \times M$ complex matrix.

From (14) we can get

$$\mathbf{R}_1 = \begin{bmatrix} \mathbf{a}^H(\beta_1) \\ \mathbf{a}^H(\beta_2) \\ \vdots \\ \mathbf{a}^H(\beta_P) \end{bmatrix}^+ \begin{bmatrix} y_{11} & y_{12} & \cdots & y_{1K} \\ y_{21} & y_{22} & \cdots & y_{2K} \\ \vdots & \vdots & \ddots & \vdots \\ y_{P1} & y_{P2} & \cdots & y_{PK} \end{bmatrix} [\mathbf{a}(\gamma_1)\ \mathbf{a}(\gamma_2)\ \cdots\ \mathbf{a}(\gamma_K)]^+ \quad (15)$$

$P \geq M$ and $K \geq M$ are required to obtain a unique solution. $[\bullet]^+$ represents Moore-Penrose pseudo-inverse matrix operation.

As mentioned previously, $\mathbf{R}_2$ for the mobile terminal located at θ_2 can be obtained in the similar way.

B. Estimating the Gains, Phases and Mutual Coupling

In this part, we present an iteration algorithm to estimate the gains, phases and mutual coupling coefficients.

1) Theory for the algorithm

Principle of the iteration algorithm is orthogonal properties between signal subspace and noise subspace.

For a mobile terminal located at θ, from Eq. (6), the covariance matrix is given by $\mathbf{R} = \mathbf{\Gamma}^H \mathbf{C}^H \mathbf{a}(\theta)\mathbf{a}^H(\theta)\mathbf{C}\mathbf{\Gamma}$. Define the eigen values and eigen vectors of $\mathbf{R}$ are λ_i and u_i, $i = 1,2,\cdots,M$, respectively, where λ_i follows a decreasing order. According to the theory of subspace [9], we have

(1) $\lambda_2 = \lambda_3 = \cdots = \lambda_M = \sigma_n^2$

(2) $\mathbf{\Gamma}^H \mathbf{C}^H \mathbf{a}(\theta)$ is orthogonal to $\mathbf{U} = [\mathbf{u}_2, \mathbf{u}_3, \cdots, \mathbf{u}_M]$.

Define the cost function J_c as

$$\begin{aligned} J_c &= \sum_{n=1}^{2} \left\| \hat{\mathbf{U}}_n{}^H \Gamma^H \mathbf{C}^H \mathbf{a}(\theta_n) \right\|^2 \\ &= \sum_{n=1}^{2} \mathbf{a}(\theta_n)^H \mathbf{C}\Gamma\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \Gamma^H \mathbf{C}^H \mathbf{a}(\theta_n) \end{aligned} \quad (16)$$

where $\hat{\mathbf{U}}_n$ is the estimation of the noise subspace $\mathbf{U}_n$ of $\mathbf{R}_n$.

As signal subspace and noise subspace are orthogonal, algorithm can be designed to obtain $\mathbf{\Gamma}$ and $\mathbf{C}$ by minimizing J_c assuming that we know the locations of two mobile terminals as θ_1 and θ_2.

2) Iteration algorithm

The initialization includes the following steps:

(1) Set the iteration counter to zero: k＝0.
(2) Select initial values for the gain-phase matrix $\mathbf{\Gamma}$ and initial value for the mutual coupling matrix $\mathbf{C}$. Usually the initial values are based on some previous knowledge (e.g., last measured values or predictions based on idealized model).
(3) Computing $\mathbf{R}_1$ and $\mathbf{R}_2$.
(4) Perform eigenanalysis and construct noise subspace.

Three steps are involved in the procedure to minimize J_c: firstly, fix $\mathbf{C}$ and obtain $\mathbf{\Gamma}$; secondly, fix $\mathbf{\Gamma}$ and obtain $\mathbf{C}$. Then repeat the above steps until J_c converges.

a) Step 1: Estimating the gains and phases

In this step, we fix $\mathbf{C}$ and minimize J_c with respect to the gain and phase of each sensor.

$$\min_{\mathbf{\Gamma}} J_c = \min_{\mathbf{\Gamma}} \sum_{n=1}^{2} \mathbf{a}(\theta_n)^H \mathbf{C}\Gamma\hat{\mathbf{U}}_n\hat{\mathbf{U}}_n^H \Gamma^H \mathbf{C}^H \mathbf{a}(\theta_n) \quad (17)$$

Let

$\boldsymbol{\delta} = [\Gamma_{11}, \Gamma_{22}, \cdots, \Gamma_{MM}]^H$, $\mathbf{Q}_1(n) = \text{diag}\{\mathbf{C}^H \mathbf{a}(\theta_n)\}$,

$\mathbf{D} = \sum_{n=1}^{2} \mathbf{Q}_1^H(n) \hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \mathbf{Q}_1(n)$,

Then (17) can be rewritten as

$$\min_{\boldsymbol{\delta}} J_c = \boldsymbol{\delta}^H \mathbf{D} \boldsymbol{\delta} . \qquad (18)$$

Constraints for (18) are $\Gamma_{11} = 1$, $\boldsymbol{\delta}^H \mathbf{w} = 1$, where $\mathbf{w} = [1,0,0,\cdots,0]^T$, $[\bullet]^T$ represents transpose operation.

By Lagrange's theorem, the estimation of $\boldsymbol{\delta}$ can be obtained for minimized J_c in (18)

$$\hat{\boldsymbol{\delta}} = \mathbf{D}^{-1}\mathbf{w} / (\mathbf{w}^T \mathbf{D}^{-1} \mathbf{w}) . \qquad (19)$$

Then we can compute the gain-phase matrix $\boldsymbol{\Gamma}$ from the vector $\hat{\boldsymbol{\delta}}$ given by (19)

$$\boldsymbol{\Gamma} = \left[diag(\hat{\boldsymbol{\delta}})\right]^H . \qquad (20)$$

b) Step 2: Estimating the Mutual Coupling Matrix

In this step, we hold the sensor gain-phase fixed and work out the mutual coupling matrix that minimizes the cost function J_c. For a circular array, $\mathbf{C}$ is circulant, and then Lemma 2 in [3] will be used for the procedure. For a linear array, $\mathbf{C}$ is Toeplitz, and then Lemma 3 in [3] will be used for the procedure. In this paper, our derivation is based on the theory for a uniform linear array. The derivation can be extended to the application for a circular array.

The optimization problem can be formulated as

$$\min_{\mathbf{C}} J_c = \min_{\mathbf{C}} \sum_{n=1}^{2} \mathbf{a}(\theta_n)^H \mathbf{C}\boldsymbol{\Gamma}\hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \boldsymbol{\Gamma}^H \mathbf{C}^H \mathbf{a}(\theta_n) \qquad (21)$$

According to [3], let $\mathbf{Q}_3(n) = \mathbf{Q}_3(\mathbf{a}(\theta_n))$,

$$c_i = C_{1i}^H , \quad i = 1,2,\cdots,M ,$$

$$\mathbf{G} = \sum_{n=1}^{2} \mathbf{Q}_3^H(n) \boldsymbol{\Gamma}\hat{\mathbf{U}}_n \hat{\mathbf{U}}_n^H \boldsymbol{\Gamma}^H \mathbf{Q}_3(n) ,$$

Then (21) can be rewritten as

$$\min_{\mathbf{c}} J_c = \mathbf{c}^H \mathbf{G} \mathbf{c} . \qquad (22)$$

Constraints for (22) are $C_{11} = 1$, $\mathbf{c}^H \mathbf{w} = 1$, where $\mathbf{w} = [1,0,0,\cdots,0]^T$.

By Lagrange's theorem, the estimation of $\mathbf{c}$ can be obtained for minimized J_c in (22)

$$\hat{\mathbf{c}} = \mathbf{G}^{-1}\mathbf{w} / (\mathbf{w}^T \mathbf{G}^{-1} \mathbf{w}) . \qquad (23)$$

Then we can reconstruct the mutual coupling matrix $\mathbf{C}$ form the vector $\hat{\mathbf{c}}$ given by (23).

c) Step3: Iteration and convergence check

Calculate J_c^{k+1} using the estimated sensor gain / phase and mutual coupling matrix. If $J_c^k - J_c^{k+1} > \varepsilon$ (ε is a preset threshold), then update the iteration counter $k = k+1$ and go back to step 1. If $J_c^k - J_c^{k+1} \le \varepsilon$, stop.

The algorithm performs the iterations until J_c converges. Note that at each step that cost function reduces so that

$$J_c^{(0)} > J_c^{(1)} > \cdots > J_c^{(k)} \ge 0 .$$

Hence, $J_c^{(k)}$ is a convergent series and convergence is guaranteed.

IV. SIMULATION RESULTS

Simulations were carried out to evaluate performance of the calibration algorithm. Consider a uniform linear array of eight omni-directional sensors separated by half a wavelength of the actual narrow-band source signals. The noise source was additive white Gaussian noise with zero mean. The following parameters are also used in the simulations.

SNR = 20 dB, SAMPLING = 100,

beamforming direction: $\alpha = 60^\circ$,

the mobile terminals' location: $\theta_1 = 45^\circ, \theta_2 = 120^\circ$

coupling coefficients:

$c = 1, 0.2, 0.09, 0.06, 0.05, 0.03, 0.02, 0.01$,

gains: $g = 1, 1.0351, 0.9886, 1.0471, 1, 1.0233, 1.1614, 1$

phases: $\phi = 0^\circ, -215^\circ, -149^\circ, -14^\circ, 21^\circ, 3^\circ, -40^\circ, -40^\circ$

where the gains and phases were measurement data from a smart antenna test bed in Xi'an Jiaotong University.

Fig. 3 and Fig. 4 show comparison between the beamforming pattern without calibration, with calibration and the ideal pattern. It is clear that without calibration the beamforming direction is wrong and the power difference between main lobe and side lobe is small. From Fig. 4, we can see that after calibration the beamforming pattern is similar as the ideal pattern. The direction of beamforming is correct and the power of main lobe is about 15 dB higher than the power of side lobe.

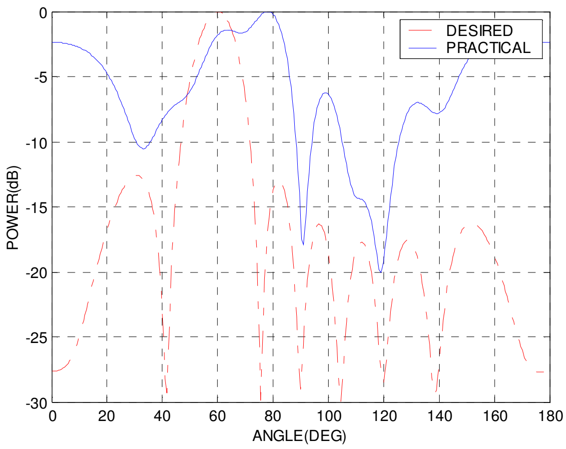

Fig. 3 Ideal beamforming pattern and the pattern without calibration, beamforming direction: $\alpha = 60^\circ$.

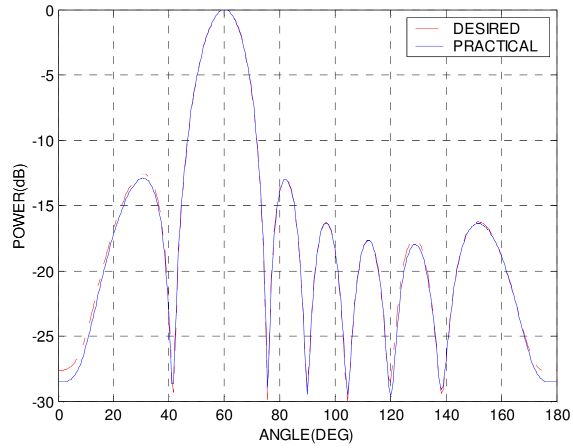

Fig. 4 Ideal beamforming pattern and the pattern with calibration, beamforming direction: $\alpha = 60^\circ$.

Fig. 5 shows the reduction of the cost function value during the iterations, until convergence is obtained.

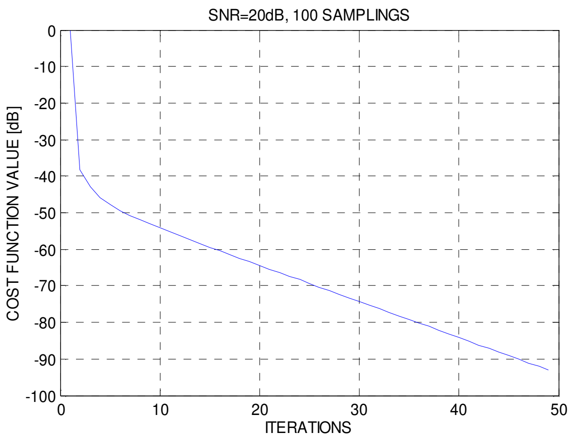

Fig. 5 Value of the cost function J_c vs. iterations.

Fig. 6 shows the relative estimation errors for gain / phase versus iterations. We can see that initially the relative errors are above 70% and after 20 iterations the relative errors reduce to less than 5%.

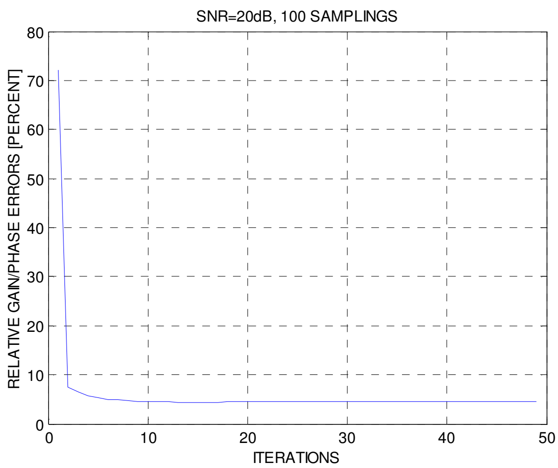

Fig. 6 Relative estimation errors for gain / phase vs. iterations.

Fig. 7 shows the relative estimation errors for coupling coefficients vs. iterations. We can see that before the first iteration the error is high and it reduces to 2%.

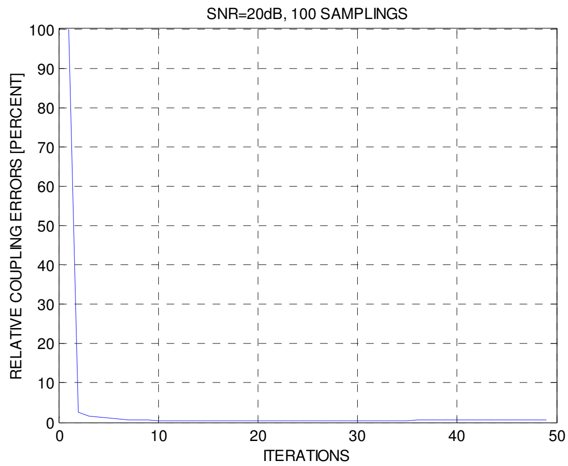

Fig. 7 Relative estimation errors for coupling coefficients vs. iterations.

V. Conclusion

We proposed an approach based on the eigenstructure analysis to obtain estimates of sensor gain, phase and mutual coupling coefficients in smart antenna downlink systems. The calibration procedure was presented. Simulation results with data collected from a smart antenna test bed showed that the good performance of downlink beamforming was obtained after calibration and the estimation errors for gain, phase and mutual coupling coefficients were less than 5%.

References

[1] F. Gross, *Smart Antennas for Wireless Communications.* McGraw-Hill Professional, 2005.

[2] J. Liberti, and T. S. Rappaport, *Smart Antennas for Wireless Communications: IS-95 and Third Generation CDMA Applications.* Prentice Hall, 1999.

[3] B. Friedlander and A. J. Weiss, “Direction finding in the presence of mutual coupling”, *IEEE Trans. Antennas Propagat.,* vol. 39, No.3, March 1991, pp.273-284.

[4] R. B. Ertel, Z. Hu, and J. H. Reed, “Antenna array hardware amplitude and phase compensation using baseband antenna array outputs”, in *Proc. 49th IEEE Vehicular Technology Conference*, 1999, vol. 3, pp. 1759–1763.

[5] K. R. Dandekar, H. Ling, and G. Xu, “Smart antenna array calibration procedure including amplitude and phase mismatch and mutual coupling effects”, in *Proc. IEEE International Conference on Personal Wireless Communications*, 2000, pp. 293–297.

[6] N. Itou, Y. Inoue, and H. Arai, “Array antenna calibration by antenna switching system”, in *Proc. IEEE Antennas and Propagation Society International Symposium*, 2005, vol. 1A, pp. 222–225.

[7] S. Hyeon, Y. Yun, and S. Choi, “Novel automatic calibration technique for smart antenna systems”, *Digital Signal Processing*, 2007, Elsevier Inc.

[8] B. Friedlander, “A sensitivity analysis of the MUSIC algorithm”，*IEEE Trans. Acoust., Speech, Signal Progressing*, vol. 38, Oct. 1990, pp.1740-1751.

[9] A. Bojanczyk, and G. Cybenko, Ed., *Linear Algebra for Signal Processing*, Springer, 1995.